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AUTHOR Thompson, Bruce
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ABSTRACT

Canonical correlation (CC) analysis is discussed with a view toward providing an intuitive understanding of how the technique operates. CC analysis entails calculation of one or more sets of canonical variate coefficients (CVC), i.e., weights which can be applied to the variables in a study. A canonical function (CF) always consists of exactly two canonical variates calculated so that the product-moment correlation between them is maximized. Thus, a squared CC coefficient indicates the proportion of variance shared by two sets of variables which each have been weighted by variate coefficients so that the CC will be as large as possible. The number of CFs which can be derived for a given data set is equal to the number of variables in the smaller of the two variable sets. CC analysis actually involves analysis of a matrix which is computed from the inter-variable correlation matrix and is appropriately applied when three assumptions are met. These assumptions are discussed, an heuristic application of CC analysis is used to clarify how the procedure operates, and four additional coefficients which greatly aide interpretation efforts are defined. Interpreting canonical results is discussed from each of three levels of specificity. (RL)

CANONICAL CORRELATION:

RECENT EXTENSIONS FOR MODELLING EDUCATIONAL PROCESSES

Bruce Thompson

University of New Orleans 70122

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Canonical correlation is a sophisticated multivariate technique which can be used to study relationships between two variable sets which each contain more than one variable. Although the procedure has been available for several decades (Hotelling, 1935), relatively few researchers have used the technique in published studies, despite the fact that "some research problems almost demand canonical analysis" (Kerlinger, 1973, p. 652). One reason why the technique is rarely used involves the difficulties which can be encountered in trying to interpret canonical results. However, these difficulties can be greatly reduced if the procedure is correctly implemented.

In this chapter, canonical correlation analysis is discussed with a view toward providing an intuitive understanding of how the technique operates. Some relatively new canonical coefficients will also be discussed.

X.1: Overview of the Procedure

Canonical correlation analysis entails the calculation of one or more sets of *canonical variate* coefficients, i.e.-- weights which can be applied to the variables in the study. Each set of canonical variate coefficients constitutes a *canonical function*. Thus, each function consists of v variate coefficients, where v is the total

number of variables in the analysis. On a given canonical function, the variate coefficients associated with the two variable sets each constitute a canonical variate. Thus, a canonical function always consists of exactly two canonical variates.

The canonical variate coefficients are calculated so that two criteria will both be met. For each function, the variate coefficients are calculated so that the product-moment correlation, i.e.-- the *canonical correlation* ($\underline{R}_c$), between the two sets of variables is maximized. In fact, for any one function, no other canonical variate coefficients which will result in a larger $\underline{R}_c$ can possibly be identified. Thus, a squared canonical correlation coefficient, $\underline{R}_c^2$, indicates the proportion of variance shared by two sets of variables which each have been weighted by variate coefficients so that $\underline{R}_c$ will be as large as possible. It is important to emphasize, however, that $\underline{R}_c$ does not represent the amount of variance which the unweighted, i.e.-- the "original," variables shared. The canonical correlation coefficient, like the multiple correlation coefficient, can range in value between 0.0 and +1.0 inclusive. A second restriction on the calculation of the variate coefficients is that the product-moment correlations between all pairs of canonical functions must be zero. In other words, each canonical function is always

perfectly uncorrelated with every other function identified in an analysis.

The number of canonical functions which can be derived for a given data set is equal to the number of variables in the smaller of the two variable sets. Of course, some or all of the computed canonical functions may be statistically non-significant.

Canonical correlation analysis actually involves analysis of a matrix which is computed from the inter-variable correlation matrix. Consequently, the analysis typically begins with computation of the correlation matrix, R. This matrix is symmetric, i.e.-- the number of rows in the matrix equals the number of columns in the matrix, and is of the order _v_ by _v_. Because the analysis actually operates by manipulating the correlation matrix, if a researcher has access to a correlation matrix, a canonical correlation analysis can be conducted without access to the original data which were used to calculate R.

Canonical correlation analysis is appropriately applied when three assumptions are met. First, as should now be apparent, the technique requires that "true" correlations among the original variables can indeed be computed. This does not mean, however, that all the variables in a study must be measured at the interval level of scale. As noted

earlier, several correlation coefficients have been shown to be algebraically equivalent to the product-moment formulation, and any of these indices may appropriately be examined in a canonical analysis. Cooley and Lohnes (1976, p. 209) have provided an exa ple application of the tecnhique in a situation where the variables in a study were not all measured at the interval level of scale.

The second assumption of canonical correlation analysis is that the magnitude of the coefficients in the correlation matrix must not be attenuated by large differences in the shapes of the variables' distributions. The impacts of distribution shapes on correlation coefficients has been discussed previously. It is important to remember, however, that the entries in a correlation matrix can only approach the extremes of the index (-1.0 to 1.0) when the variables are similarly distributed. Of course, if the entries in the correlation matrix have been attenuated by disparities in the distributions of the original variables, this will necessarily affect the canonical analysis of the matrix which the procedure derives from R.

A third assumption must be met when the researcher employs test statistics to determine the statistical significance of results. As noted previously, multivariate test statistics require an assumption that the variables

have a multivariate normal distribution in the population. This distribution is the multivariate analogue to the bivariate normal distribution. The characteristics of the multivariate normal distribution will not be discussed here.

One reason for limited discussion of the distribution is that there is not an accepted test statistic which can be applied to sample data to estimate the probability that any deviations from multivariate normality may involve sampling error. This suggests a paradox. If the researcher knows that the population distributions are multivariate normal, the researcher probably also knows the other population parameters, and would not need to apply any test statistics in the first place. Alternatively, if the researcher is not certain that the population distributions are multivariate normal, little can be done to resolve this uncertainty short of gathering data from the entire population of interest.

Therefore, researchers will generally be uncertain as to whether or not they have met the distribution assumption of canonical correlation analysis. It is important to emphasize, however, that examining the univariate or bivariate distributions of the sample data will not help to resolve this uncertainty. Multivariate distributions can be non-normal even when all subsets of univariate or bivariate distributions are normally distributed. This was implied in

the previous demonstration that a bivariate normal distribution is not necessarily always created by joining together two variables which both are individually distributed in a univariate normal fashion.

A second reason for limited discussion of the characteristics of the multivariate normal distribution is also noteworthy. There is a basic theorem in mathematical statistics called the multivariate central limit theorem. This theorem suggests that when sample size is "large," certain indices derived from original variables will tend to be normally distributed, even when the original variables were not themselves distributed in a multivariate normal manner. This assurance can be relied upon to justify the application of test statistics in a canonical correlation analysis when the researcher's sample is large. However, there is no generally accepted rule for determing when a sample is big enough to be considered "large." Consequently, it is generally desirable to obtain the largest possible sample size, since this tends to insure that the central limit theorem can legitimately be invoked as a justification for applying test statistics.

X.2: An Heuristic Application

An heuristic application of canonical correlation analysis may clarify how the procedure operates, and will provide a framework for further discussion. A study by Thompson (1979) will be discussed for these purposes.

The criterion variables in the study were teachers' preferences for various models of teaching. The author argued that it is important to understand teachers' instructional method preferences, because these preferences may ultimately affect teaching practices and the learning of children. The predictor variables in the study were teachers' preferences for various educational philosophies, and the teachers' perceptions regarding which characteristics distinguish "best" from other teachers. This last variable was labelled "role-ideals." In essence, the author argued that the criterion and the predictor variables should be related, because "educational procedures are generated from general views about human nature and about the kinds of goals and environments that enhance human beings" (Joyce and Weil, 1972, p. 5). The null hypothesis in the study was that there would be no statistically significant canonical correlation between the two variable sets (α=.05). The sample consisted of 235 teachers.

The study examined relationships among 14 variables. The variables were the scores of the 235 subjects on the factors which are briefly described in Table X.1. The role-ideals factors were discussed in greater detail in the chapter about factor analysis.

Insert Table X.1 about here.

As noted previously, canonical correlation analysis always begins with the computation of a correlation matrix. The correlation matrix which was computed in the study is presented in Table X.2. The reader may wonder why the matrix is partitioned into different portions in the table. Canonical correlation analysis actually requires that R be partitioned just as it has been in Table X.2. The section of the matrix which summarizes intra-domain relationships involving only the criterion variables is denoted R_{11}, and is of the order $\underline{y}$ by $\underline{y}$, where $\underline{y}$ represents the number of the criterion variables in the study. In this case, $\underline{y}$ equals 4. The section of the matrix which summarizes intra-domain relationships involving the predictor variables is denoted R_{22}, and is of the order $\underline{x}$ by $\underline{x}$, where $\underline{x}$ represents the number of the predictor variables in the study. In this case, $\underline{x}$ equals 10. The remaining two matrix partitions are denoted R_{12} and R_{21}, and are respectively of the order $\underline{y}$ by $\underline{x}$ and $\underline{x}$ by $\underline{y}$.

Insert Table X.2 about here.

After the matrix, R, has been partitioned, the different partitions are then used to compute a new matrix. The details of this operation will not be presented here (see Cooley and Lohnes, 1971, pp. 176-179). Suffice it to say that the required mathematics always produce results which meet the two previously specified criteria, i.e.--maximum canonical correlations and perfectly uncorrelated canonical functions. In this case the analysis produced the results presented in Table X.3.

Insert Table X.3 about here.

X.3: Four Additional Coefficients

Several computer statistics "packages" will compute the coefficients presented in Table X.3. However, four additional sets of canonical coefficients can greatly aide interpretation efforts, and can either be calculated by hand or with the assistance of non-commericial computer programs (cf. Thompson and Frankiewicz, 1979).

A *structure coefficient* (Cooley and Lohnes, 1971) indicates the degree of correlation between an original variable and the variate defined by the variate coefficients of all the variables in the same domain. The square of a structure coefficient is the proportion of variance that an original variable linearly shared with a canonical variate. For any one function, the structure coefficients for the

criterion variate can be computed by multiplying the R_{11} matrix times the criterion variate coefficients. Similarly, the predictor variable structure coefficients can be computed by multipling the R_{22} matrix times the predictor variate coefficients. This formula implies that variate and structure coefficients for the variables in a domain will be equivalent only when the original variables in the domain are perfectly uncorrelated. Conceptually, structure coefficients are bivariate correlation coefficients, and can range in value between -1.0 and +1.0 inclusive.

Canonical *variate adequacy* coefficients indicate how "adequately" a canonical variate represented the variance of the original variables in a domain. When a variate captures 100% of the variance of the original variables, the adequacy coefficient will equal 1.0. The smallest value which can be attained by a variate adequacy coefficient is .0, although this value would only be observed on a hypothetical function for which $\underline{R}_c$ exactly equaled .0.

Canonical *redundancy coefficients* (Stewart and Love, 1968) indicate the proportion of variance shared by two variates ($\underline{R}_c^2$) which is contained in one variate and which is linearly redundant to the variance in the original variables of the other domain. Redundancy coefficients can be important to the interpretation of canonical correlation

analysis results, but are especially important when the number of variables studied is large relative to the number of subjects in a study. In these instances the canonical correlation coefficient has a strong positive bias. Fortunately, the redundancy coefficient is less biased, and Miller (1975) has developed a distribution theory which enables the researcher to test the statistical significance of these coefficients.

Mathematically, the redundancy coefficient for the criterion variables, $\underline{Rd}_Y$, equals the adequacy coefficient of the predictor variate times $\underline{R}_c^2$. Conversely, the redundancy coefficient for the predictor variables, $\underline{Rd}_X$, equals the adequacy coefficient for the criterion variate times $\underline{R}_c^2$. This formula indicates that a redundancy coefficient can be as large as 1.0, since $\underline{R}_c^2$ can theoretically be 1.0, and since structure coefficients can also theoretically each equal 1.0. Since adequacy coefficients and $\underline{R}_c^2$ both can be no smaller than 0.0, 0.0 is also the lower bound for redundancy coefficients. The mathematics of this calculation may now be clear, but the psychometric interpretation of the coefficient is complicated, and will merit more complete discussion momentarily.

Canonical *index coefficients* were apparently first discussed by Frankiewicz and Merrifield (1967). A canonical index coefficient indicates the correlation between an original variable and the variables in the other domain, once the variables in the other domain have been weighted by their canonical variate coefficients. The square of a canonical index coefficient represents the proportion of variance which an original variable linearly shared with the variate in the other domain.

Index coefficients can be calculated in either of two mathematically equivalent ways. The first method is analagous to the computation of canonical structure coefficients. The index coefficients for the criterion variables equals the R_{12} inter-domain correlation matrix times the variate coefficients for the predictor variables. The index coefficients for the predictor variables equals the R_{21} inter-domain correlation matrix times the variate coefficients for the criterion variables. An equivalent procedure can be performed more conveniently by hand, since the second procedure does not entail matrix algebra. The index coefficient for a variable is equal to the variable's structure coefficient times $\underline{R}_c$. Index coefficients are similar to bivariate correlation coefficients, and can range in value between -1.0 and +1.0 inclusive.

Index coefficients have some noteworthy interpretations. The sum of the squared index coefficients for all the variables in one domain, divided by the number of variables in the domain, will equal the other domain's redundancy coefficient. Thus, if all the index coefficients for the variables in a set are squared and then divided by the number of variables in the domain, and these results are then each divided by the redundancy coefficient for the other domain, the proportional contribution of each original variable toward creating redundancy can be determined.

Also, when the variables in a domain are perfectly uncorrelated with each other, the sum of the squared index coefficients for the variables in a domain will equal $\underline{R}_c^2$. When the variables in a domain are correlated, the sum of the squared index coefficients for a domain will be greater than $\underline{R}_c^2$, because the original variables overlap and the squared index coefficients in part make non-unique contributions toward the definition of $\underline{R}_c^2$. However, whether or not the original variables in a domain are uncorrelated, the contribution which each original variable made toward defining a canonical correlation can be directly evaluated as a proportion, i.e.-- by dividing each squared index coefficient by $\underline{R}_c^2$.

X.4 The Psychometrics of Rd

The behavior and meaning of redundancy coefficients warrant special attention. The first revelation in particular will seem counterintuitive. The redundancy coefficients for the two variable sets do not have to be equal. In fact, they rarely will be equal. A figurative analogy may facilitate understanding of why $\underline{Rd}_Y$ need not equal $\underline{Rd}_X$.

Suppose that a researcher wanted to explore the nature of relationships between criterion variables involving human behavior, and predictor variables which measure human psychology. Assume that the essence of human behavior could be well represented by measuring four criterion variables: caloric intake per day, dollars saved each week, dating behavior, and the political behaviors involved in voting for various candidates. Assume that the essence of human psychology could be well represented by measuring three predictor variables: strength of drive to satisfy hunger, strength of drive to have shelter, and strength of the desire for sex.

After the data were collected, the researcher might conduct a canonical correlation analysis. As many as three canonical functions might be isolated, since the smaller of the two variable sets consisted of three variables. Conceivably, one function might define a canonical

relationship which involved the behavior variables primarily represented by the dollars saved variable, as indicated by the calculated structure coefficients. The psychology variables may have been mainly represented by the hunger drive variable.

Two additional assumptions are also required. Assume the researcher knows that the dynamics of human behavior are well represented by the dollars saved variable. However, assume too that the researcher knows that the dynamics of human psychology are not very adequately represented by the hunger drive variable. In such a case, $\underline{Rd}_X$ would be greater than $\underline{Rd}_Y$. In other words, the new variance gained by adding the behavior variate's variance to the variance embedded in an already available psychology variate would be greater than the variance gained by adding the psychology variate's variance to the variance embedded in an already available behavior variate.

It is important to understand what causes redundancy coefficients to not be equal. Canonical variate coefficients, it should be remembered, are calculated both to represent the variance of the original variables and to maximize $\underline{R}_C$. The second consideration will always outweight the first consideration if this will serve to increase $\underline{R}_C$. When the first consideration is differentially met with

respect to one of the two variable sets, in order that a larger $\underline{R}_c$ can be computed, the redundancy coefficients will not be equal.

This conclusion forces a closer examination of the psychometric meaning of redundancy coefficients. A redundancy coefficient can equal 1.0 only when two variates share exactly 100% of their variance ($\underline{R}_c^2$=1.0) and a variate perfectly represents the original variables in its domain, i.e.-- all the squared structure coefficients for the domain equal 1.0. This suggests that redundancy coefficients can be interpreted as indices which assess how well intra-domain relationships are represented by g-variates and how well inter-domain relationships are represented by g-functions.

Canonical structure, adequacy, redundancy, and index coefficients for the study are presented in Table X.4. The tabled coefficients may be examined to verify computational procedures, and to determine if the coefficients are inter-related in the indicated manner.

<u>Insert Table X.4 about here.</u>

X.5 Interpreting Canonical Results

The neophyte student of canonical correlation analysis may be overwhelmed by the myriad coefficients which the procedure produces. This implies a strength of the procedure. Canonical correlation analysis produces results

which can be theoretically rich, and if properly implementd the procedure can adequately capture some of the complex dynamics involved in educational reality. Interpretation of the results may clarify the manner in which canonical correlation results can be interpreted. Interpretation can be couched at each of three levels of specificity.

First, the results may be examined at a global level. For example, the magnitude of the observed variate overlaps ($\underline{R}_c^2$) may be tested for statistical significance, if the researcher collected sample rather than population data, and if the researcher has reason to believe that the sample was representative of the population of interest. The statistical significance of a canonical correlation can be assessed by applying a chi-square test statistic. The computation of the "calculated" chi-square will not be presented here (see Tatsuoka, 1971, pp. 188-189), since the computation is typically performed by computer statistics packages and such a discussion would shed little light on the meaning of $\underline{R}_c$. It is important to understand how the degrees of freedom for the test statistic are computed, however, and these computations are illustrated in Table X.5. Table X.5 also presents the $\underline{R}_c$'s, the "calculated" test statistics, the degrees of freedom for the "calculated" test statistics, the "critical" test statistics, and the decisions associated with each of the four calculated

canonical functions. The test statistics evaluate the null hypothesis that a given population $\underline{R}_c$ equals .0.

<u>Insert Table X.5 about here.</u>

The Table X.5 results indicate that, assuming the null hypotheses to be true and the sample to be representative, canonical correlations of the magnitude of the four obtained correlations would have been obtained in less than 5% of the random samples from a population in which the canonical correlations were actually each equal to 0.0. Thus, all four of the canonical correlations were "statistically significant." However, since the calculated probabilities are sensitive to sample size, the researcher should be particularly attentive to the "educational significance" of obtained results. The educational significance of canonical correlation results can partly be assessed by examining how much variance the sets of weighted original variables shared with each other. However, there is no absolute criterion regarding when a $\underline{R}_c^2$ suggests that a relationship is educationally important. These decisions necessarily involve professional judgment and will vary from one study to another. In this study, the overlaps of the weighted original variables ranged in value from 21.1% ($\underline{R}_c^2$ X 100) to 9.4%.

The results can also be interpreted at an aggregate level by calculating communality coefficients for the variables. A communality coefficient indicates the proportion of an original variable's variance which can be reproduced from the canonical variates calculated for its domain, and is equal to the sum of all the variable's squared structure coefficients. In this case, 100% of the variance of each of the four criterion variables could be reproduced from the four criterion variates.

Examination of the communalaties of the predictor variables suggests another important conclusion. The communalities for the Rationalism ($\underline{h}^2$=.19) and the humanism ($\underline{h}^2$=.22) variables were not very large. This suggests that a more parsimonious analysis could be achieved by deleting at least these two variables from a re-analysis.

A third global interpretation of results can be accomplished by examining the redundancy coefficients. In this case, g-functions and g-variates were not anticipated. Thus the fact that the redundancy coefficients suggest the absence of g-functions and g-variates has little theoretical meaning in this study.

A second level of interpretation involves examining the results at their most specific level. This generally can best be accomplished by examining the canonical structure

coefficients. In this study all the variables were scored so that high scores were associated with preference for a model of teaching, or agreement with a philoso y statement, or agreement that ideal teachers possess a characteristic. Consequently, when a set of structure coefficients all were positive this meant that preference for the involved model of teaching, or agreement with a philosophy or an ideals description, tended to go together. Alternatively, when a set of structure coefficients all were negative, this meant that dislike for the involved model of teaching, or disagreement with the philosophy or ideals description, tended to go together.

Four distinct patterns were identified in the study:

1. Teachers who ascribe characteristics of simpleness to non-ideal teachers, agree with the tenets of Progressivism, and are anti-Existentialist in outlook, tend to prefer to teach with models of teaching which emphasize incisive understanding of a discipline's content and methodology.
2. Teachers who ascribe characteristics of simpleness and warmth to their role-ideals tend to prefer to teach with models of teaching which are affectively oriented and which emphasize inquiry methods.
3. Teachers who disagree with the tenets of Essentialism tend to dislike highly structured

models of teaching and prefer to teach with inquiry strategies.

4. Teachers who ascribe characteristics of rigor to their role-ideals, disagree with the tenets of Progressivism, and agree with the tenets of Perennialism, tend to dislike affectively oriented models of teaching, but prefer models of teaching which are highly structured.

Of course, the four patterns are perfectly uncorrelated with each other, since the functions which were interpreted to produce them were perfectly uncorrelated with each other. It is important to note, however, that the four patterns were not equally strong, since the squared canonical correlations associated with the functions were not identical.

The results can also be interpreted at a third, "moderate" level of specificity. This level of focus is appropriate in this study, because the predictor variable set itself included two conceptually different subsets of variables. One might question whether either the philosophy or the role-ideals variables accounted for a preponderant proportion of the variance linearly shared across the two variable domains. Examination of the squared canonical index coefficients could provide an answer to this question.

As noted earlier, squared canonical index coefficients indicate the proportion of variance which an original variable linearly shared with the linearly weighted variables in the other domain. These values can be summed within variable subsets, and then the sums can be compared to determine if one of the subsets contained a preponderant proportion of the variance shared across the domains. In this case, the ratios of the philosophy variable sums to the role-ideals variable sums were respectively: 1.3:1, 0.5:1, 2.0:1, and 1.1:1. These ratios suggest that the philosophy variables may have accounted for somewhat more of the variance which the weighted predictor variables shared with the weighted criterion variables. However, this result must be interpreted cautiously. Because the philosophy variables were not perfectly uncorrelated with each other, their index coefficients each contained some non-unique variance, and therefore the sums of the squared index coefficients for the philosophy variables were artificially inflated to some extent. Nevertheless, the results suggest that philosophical orientations may influence teachers more than is commonly recognized.

Overall, the results suggest that the relationships between the two domains are systematic, but the squared canonical correlation coefficients indicate that the relationships are of moderate magnitude. The criterion

variables were adequately explained by the predictor variables, as indicated by the adequacy coefficients for the criterion variates. However, the predictor variates' adequacy coefficients and the squared canonical correlation coefficients suggest that other predictor variables might share more of their variance with the criterion variables, and provide a more parsimonious explanation of teachers' preferences for models of teaching. Consequently, the results represent only an initial first step, albeit an apparently fruitful first step, toward achieving a more complete understanding of teachers' instructional method preferences.

X.6 Summary

When certain assumptions are met, canonical correlation analysis can provide a theoretically rich representation of complex educational realities. In general, it is important to obtain large sample sizes when the technique is employed. Several coefficients can provide the basis for interpretation of the results. The coefficients which are most appropriately examined in a given study will depend upon the nature of the study's variables and the study's theoretical framework.

Exercises for Additional Insight

1. It was previously explained that the multiple correlation coefficient, R, can be computed from the correlation matrix, R. Using the matrix presented in Table X.2 compute the R between each of the four criterion variables (analyzed separately) and the 10 predictor variables. Add the four squared multiple correlation coefficients together and compare the sum with the squared canonical correlations. What does this result suggest about the similarities between multiple correlation regression analysis and canonical correlation analysis? What are the necessary and sufficient conditions for comparision results to be identical to those obtained in this comparison?
2. Canonical correlation analysis is conceptually similar to factor analysis. This suggests that canonical functions could be rotated in order to improve their interpretability. What aspect of canonical correlation analysis differentiates it from factor analysis and suggests that the decision to rotate the functions should be cautiously taken? (See R.M. Thorndike. Strategies for rotating canonical components. Paper presented at the annual meeting of the American Educational Research Association, 1976. ED 123 259)
3. The similarities between canonical correlation analysis

and factor analysis may suggest that canonical function scores might be computed in a manner analagous to the computation of factor scores. How might the results of the Thompson (1979) study have been altered if an alternative analytic strategy had been followed? First, the researcher performed a canonical correlation analysis of the Table X.2 matrix. However, this time the philosophy variables were treated as criterion variables and the role-ideals variables were treated as predictor variables. After functions were computed, canonical function scores were computed for the 235 subjects in the study. Of course, this last step would require access to the original data; function scores can not be computed if only R is available. Next, the function scores were used in a second canonical correlation analysis in which the predictor variables were as before, but the criterion variables were the canonical function scores derived from the preliminary canonical analysis.

4. Canonical correlation analysis may capitalize on measurement error when attempting to maximize $\underline{R}_c$. How might the stability of canonical functions be estimated? (Hint: How was the stability of a regression equation equation estimated in the chapter on multiple regression?)

Example Applications of the Technique

Hopkins, G., Payne, D.A., & Ellett, C.D. Life history antecedents of current personality traits of gifted adolescents. *Measurement and Evaluation in Guidance*, 1975, *8*, 29-36.

Thompson, B., & Rucker, R. Two-year college students' goals and program preferences. *Journal of College Student Personnel*, in press.

Walberg, H.J. Predicting class learning: An approach to the class as a social system. *American Educational Research Journal*, 1969, *4*, 529-542.

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Walberg, H.J. Predicting class learning: An approach to the class as a social system. _American Educational Research Association_, 1969, _4_, 529-542.

TABLE X.1
Variable Descriptions

Criterion Variables:	Models of Teaching Factors
Inquiry Strategies	Models which emphasize inquiry methods, e.g.-- Thelen and Dewey's Group Investigation model
Incisive Understanding	Models which emphasize understanding of a discipline's core concepts and methods
Affective Orientation	Models which emphasize affect, e.g.-- the Awareness Training model
Structured Environment	Models which emphasize (or de-emphasize) structure, e.g.-- the Behavior Modification model
Predictor Variables:	Educational Philosophy & Role-Ideals Factors
Essentialism	There are are certain essential facts which education must emphasize.
Humanism	People should be the focus of existence and should be treated with dignity.
Perennialism	Human nature is unchanging and is the same everywhere.
Progressivism	Problem-solving skills are more important than the inculcation of content.
Rationalism	Rationality distinguishes man from animal and is the key to survival.
Existentialism	Each person, by choosing, creates reality. People must be allowed to choose.
Warm	Role-ideals who display exceptional concern or lack of concern for people as people.
Scholarly	Role-ideals who display exceptional intellect or lack of intellect and scholarliness.
Rigorous	Role-ideals who are especially exacting or who are the antithesis of being exacting.
Simple	Role-ideals who are distinguished by being simple and docile, or the opposite thereof.

TABLE X.2
Correlation Matrix

Variable	IS	IU	AO	SE	Es	H	Pe	Pr	R	Ex	W	Sc	R	Si
Inquiry Strategies	1.00	.00	.00	.00	-.15	.16	.05	.09	.07	-.10	.03	.06	.22	.13
Incisive Understanding	.00	1.00	.00	.00	.05	.08	.08	.26	.15	-.13	.05	.20	.06	-.26
Affective Orientation	.00	.00	1.00	.00	-.10	.08	-.13	.15	-.07	.16	.20	.01	-.09	.18
Structured Environment	.00	.00	.00	1.00	.17	-.02	.10	-.04	.04	.12	.12	.09	.10	.15
Essentialism	-.15	.05	-.10	.17	1.00	-.00	-.00	-.00	.00	-.00	-.01	.02	.08	.01
Humanism	.16	.08	.08	-.02	-.00	1.00	.00	.00	-.00	.00	.05	.10	.05	.04
Perennialism	.05	.08	-.13	.10	-.00	.00	1.00	.00	.00	-.00	-.14	.14	.15	-.08
Progressivism	.09	.26	.15	-.04	-.00	.00	.00	1.00	.00	.00	.13	.12	-.06	-.07
Rationalism	.07	.15	-.07	.04	.00	-.00	.00	.00	1.00	-.00	-.04	.07	.07	-.06
Existentialism	-.10	-.13	.16	.12	-.00	.00	-.00	.00	-.00	1.00	-.14	-.14	-.01	.25
Warm	.03	.05	.20	.12	-.01	.05	-.14	.13	-.04	-.14	1.00	.00	.00	.00
Scholarly	.06	.20	.01	.09	.02	.10	.14	.12	.07	-.14	.00	1.00	.00	.00
Rigorous	.22	.06	-.09	.10	.08	.05	.15	-.06	.07	-.01	.00	.00	1.00	.00
Simple	.13	-.26	.18	.15	.01	.04	-.08	-.07	-.06	.25	.00	.00	.00	1.00

NOTE: "-.00" represents values which were less than 0.0 and did not round to at least -.01.

TABLE X.3
Canonical Variate Coefficients

Variable	Function I	II	III	IV
Inquiry Strategies	-.38	-.66	.49	.43
Incisive Understanding	-.87	.03	-.42	-.25
Affective Orientation	.25	-.70	-.17	-.65
Structured Environment	.18	-.28	-.75	.57
Essentialism	.07	.32	-.62	.24
Humanism	-.23	-.29	.20	-.10
Perennialism	-.09	.03	-.21	.40
Progressivism	-.48	-.32	-.05	-.41
Rationalism	-.29	-.03	-.13	.19
Existentialism	.35	-.13	-.50	-.25
Caring	.12	-.41	-.49	-.11
Scholarly	-.23	-.13	-.40	.02
Exacting	-.30	-.29	.14	.52
Simple	.40	-.60	.17	.38

TABLE X.4
Additional Canonical Coefficients

Variables	S	I	S	I	S	I	S	I
Inquiry Strategies	-.38	-.17	-.66	-.28	.49	.16	.43	.13
Incisive Understanding	-.87	-.40	.03	.01	-.42	-.14	-.25	-.08
Affective Orientation	.25	.11	-.70	-.29	-.17	-.06	-.65	-.20
Structured Environment	.18	.07	-.28	-.12	-.75	-.25	.57	.18
Essentialism	.04	.00	.30	.12	-.61	-.20	.29	.09
Humanism	-.25	-.11	-.36	-.15	.15	.05	-.06	-.02
Perennialism	-.22	-.10	.08	.03	-.19	-.06	.46	.14
Progressivism	-.50	-.23	-.34	-.14	-.18	-.06	-.48	-.14
Rationalism	-.36	-.16	-.01	-.00	-.14	-.05	.21	.06
Existentialism	.47	.22	-.20	-.08	-.34	-.11	-.15	-.04
Warm	.03	.00	-.46	-.19	-.38	-.13	-.20	-.06
Scholarly	-.39	-.18	-.17	-.07	-.37	-.13	.07	.02
Rigorous	-.31	-.14	-.26	-.11	.07	.02	.64	.19
Simple	.54	.25	-.62	-.26	.07	.02	.30	.09
Criterion Adequacy	.25		.25		.25		.25	
Predictor Adequacy	.13		.11		.08		.11	
Redundancy of Criterion	.05		.04		.03		.02	
Redundancy of Predictor	.03		.02		.01		.01	

NOTE: "S"=structure coefficient; "I"=index coefficient.

TABLE X.5
Test Statistics

R	Chi-square Calculated	df	df Calculation	Critical Chi-square	Decision
.46	147.38	40	(4+1-1)(10+1-1)	55.76	Reject
.42	93.61	27	(4+1-2)(10+1-2)	40.11	Reject
.34	49.68	16	(4+1-3)(10+1-3)	26.30	Reject
.31	22.35	7	(4+1-4)(10+1-4)	14.07	Reject

NOTE: df for ith R_c = (no. of criterion variables + 1 - i) times (no. of predictor variables + 1 - i)